STAT 400 Examples for 1.3

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The **conditional probability of A, given B** (the probability of event A, computed on the assumption that event B has happened) is

$$\mathbf{P}(\mathbf{A} \mid \mathbf{B}) = \frac{\mathbf{P}(\mathbf{A} \cap \mathbf{B})}{\mathbf{P}(\mathbf{B})} \qquad (\text{assuming } \mathbf{P}(\mathbf{B}) \neq 0).$$

Similarly, the conditional probability of B, given A is

$$\mathbf{P}(\mathbf{B} \mid \mathbf{A}) = \frac{\mathbf{P}(\mathbf{A} \cap \mathbf{B})}{\mathbf{P}(\mathbf{A})} \qquad (\text{assuming } \mathbf{P}(\mathbf{A}) \neq 0).$$

3. (continued)

The probability that a randomly selected student at Anytown College owns a bicycle is 0.55, the probability that a student owns a car is 0.30, and the probability that a student owns both is 0.10.

С	C′	
0.10	0.45	0.55
0.20	0.25	0.45
0.30	0.70	1.00
	C 0.10 0.20 0.30	C C' 0.10 0.45 0.20 0.25 0.30 0.70

 $P(B) = 0.55, P(C) = 0.30, P(B \cap C) = 0.10.$

a) What is the probability that a student owns a bicycle, given that he/she owns a car?

b) <u>Suppose a student does not have a bicycle</u>. What is the probability that he/she has a car?

- 5. (continued) Suppose
 - P(A) = 0.22, P(B) = 0.25, P(C) = 0.28, $P(A \cap B) = 0.11,$ $P(A \cap C) = 0.05,$ $P(B \cap C) = 0.07,$ $P(A \cap B \cap C) = 0.01.$

Find the following:





d) $P(B \cup C | A)$ e) $P(C | A \cup B)$

f) $P(C | A \cap B)$ g) $P(A \cap B \cap C | A \cup B \cup C)$



Multiplication Law of Probability

If A and B are any two events, then

$$P(A \cap B) = P(A) \cdot P(B \mid A)$$
$$P(A \cap B) = P(B) \cdot P(A \mid B)$$

- 8. It is known that 30% of all the students at Anytown College live off campus.
 Suppose also that 48% of all the students are females. <u>Of the female students</u>, 25% live off campus.
- a) What is the probability that a randomly selected student is a female <u>and</u> lives off campus?

b) What is the probability that a randomly selected student either is a female <u>or</u> lives off campus, or both?

c) What proportion <u>of the off-campus students</u> are females?

d) What proportion <u>of the male students</u> live off campus?

- **9.** Suppose that Joe's Discount Store has received a shipment of 25 television sets, 5 of which are defective. On the following day, 2 television sets are sold.
- a) Find the probability that both of the television sets are defective.

b) Find the probability that at least one of the two television sets sold is defective.

- **10.** Cards are drawn one-by-one **without** replacement from a standard 52-card deck. What is the probability that ...
- a) ... both the first and the second card drawn are $\mathbf{\Psi}$'s?
- b) ... the first two cards drawn are a \forall and a \clubsuit (or a \clubsuit and a \checkmark)?
- c) ... there are at least two \forall 's among the first three cards drawn?