

# STAT 400: Homework 02

Spring 2018, UIUC

Due: Friday, February 2, 2:00 PM

Please see the [detailed homework policy document](#) for information about homework formatting, submission, and grading.

## Exercise 1

Just before their last year at Anytown High School, the seniors hold the Senior Year Kick-Off party. Sixty percent of the students attending the party are seniors, and the rest (friends, significant others, siblings, etc.) are juniors (25%), sophomores (10%), and freshmen (5%) Unfortunately, drinking is quite common at this party; 90% of the seniors consume alcohol, so do 80% of the juniors, 50% of the sophomores, and 20% of the freshmen.

- (a) If a student at this party is drinking, what is the probability that this student is a senior?
- (b) If a student at this party is not drinking, what is the probability that this student is not a senior?
- (c) If a student at this party is not a senior, what is the probability that this student is not drinking?
- (d) What proportion of the underclassmen (freshmen and sophomores) attending the party consume alcohol?
- (e) The school administration discourages the Senior Year Kick-Off party; the principal of AHS announced that any senior attending the party will receive a week of detention. Of course, drinking is also discouraged. Find the proportion of the students at the party who either are seniors, or consume alcohol, or both.
- (f) Are events {a student at the party is a senior} and {a student at the party is drinking} independent? **Justify your answer.** *No credit will be given without proper justification.*
- (g) Are events {a student at the party is a junior} and {a student at the party is drinking} independent? **Justify your answer.** *No credit will be given without proper justification.*

## Exercise 2

A [bishop](#) is placed at random (with equal chance) on a [chess](#) board (8 x 8). A [king](#) of the opposing color is placed at random (with equal chance) on one of the remaining squares. What is the probability that the king is under attack from the bishop?

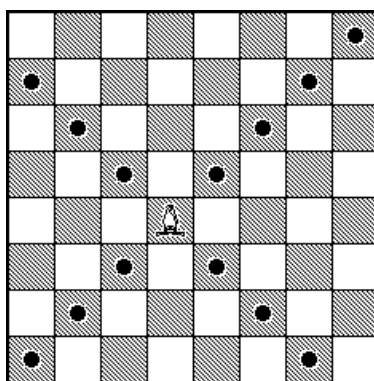


Figure 1: **Bishop:** possible attacks.

**Hint:** Placed *anywhere* on a chess board, a **rook** attacks 14 squares out of the remaining 63.

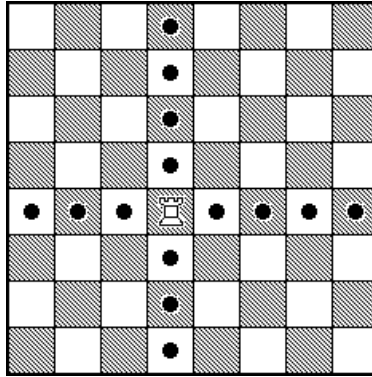


Figure 2: **Rook:** possible attacks.

$$P(\text{K is under attack}) = \frac{14}{63}$$

### Exercise 3

You are given that  $P(A) = 0.5$  and  $P(A \cup B) = 0.7$ . Student 1 assumes that  $A$  and  $B$  are independent and calculates  $P(B)$  based on that assumption. Student 2 assumes that  $A$  and  $B$  are mutually exclusive and calculates  $P(B)$  based on that assumption. Find the absolute difference between the two calculations.

### Exercise 4

Alex and David agreed to play a series of tennis games (as many as needed) until one of them wins two games in a row. Alex will serve in the first game, then the serve would alternate game by game between Alex and David. David is a better tennis player; Alex has a 50% chance of winning a game on his serve and only a 20% chance of winning a game if David serves. Assume that all games are independent. Find the probability that Alex is the first one to win two games in a row.