1. 2.3-11 $2.5-3$

If the moment-generating function of X is

$$
\mathrm{M}_{\mathrm{X}}(t)=\frac{2}{5} e^{t}+\frac{1}{5} e^{2 t}+\frac{2}{5} e^{3 t}
$$

Find the mean, variance, and pmf of X .
2. Suppose a discrete random variable $X$ has the following probability distribution:

$$
f(0)=\frac{7}{8}, \quad f(k)=\frac{1}{3^{k}}, \quad k=2,4,6,8, \ldots
$$

(possible values of X are even non-negative integers: $0,2,4,6,8, \ldots$ ).
Recall Week 02 Discussion Problem 1 (a): this is a valid probability distribution.
a) Find the moment-generating function of $\mathrm{X}, \mathrm{M}_{\mathrm{X}}(t)$. For which values of $t$ does it exist?
b) Use $\mathrm{M}_{\mathrm{X}}(t)$ to find $\mathrm{E}(\mathrm{X})$.
3. Suppose a discrete random variable $X$ has the following probability distribution:

$$
f(1)=\ln 3-1, \quad f(k)=\frac{(\ln 3)^{k}}{k!}, \quad k=2,3,4, \ldots
$$

(possible values of X are positive integers: $1,2,3,4, \ldots$ ).
Recall Week 02 Discussion Problem 1 (b): this is a valid probability distribution.
a) Find the moment-generating function of $\mathrm{X}, \mathrm{M}_{\mathrm{X}}(t)$. For which values of $t$ does it exist?
b) Use $\mathrm{M}_{\mathrm{X}}(t)$ to find $\mathrm{E}(\mathrm{X})$.
4. Suppose the moment-generating function of $X$ is

$$
\mathrm{M}_{\mathrm{X}}(t)=0.1 e^{2 t}+0.3 e^{4 t}+0.6 e^{7 t}
$$

a) Find $\mu=E(X)$.
b) Find $\sigma=\operatorname{SD}(X)$.
5. Suppose a discrete random variable $X$ has the following probability distribution:

$$
f(k)=\mathrm{P}(\mathrm{X}=k)=a^{k}, \quad k=2,3,4,5,6, \ldots, \quad \text { zero otherwise. }
$$

a) Find the value of $a$ that makes this is a valid probability distribution.
b) Find $\mathrm{P}(\mathrm{X}$ is even $)$.
c) Find the moment-generating function of $\mathrm{X}, \mathrm{M}_{\mathrm{X}}(t)$. For which values of $t$ does it exist?
d) Find $E(X)$.
6. Let X be a continuous random variable with the probability density function

$$
f(x)=\frac{C}{x^{4}}, \quad x>5, \quad \text { zero otherwise }
$$

a) Find the value of $C$ that would make $f(x)$ a valid probability density function.
b) Find the cumulative distribution function of $\mathrm{X}, \mathrm{F}(x)=\mathrm{P}(\mathrm{X} \leq x)$.
"Hint": Should be $F(5)=0, F(\infty)=1$.
c) Find the probability $\mathrm{P}(6<\mathrm{X}<10)$.
f) Find the 80th percentile of the distribution of $\mathrm{X}, \pi_{0.80}$.
g) Find the expected value of $\mathrm{X}, \mathrm{E}(\mathrm{X})$.
h) Find the standard deviation of $\mathrm{X}, \mathrm{SD}(\mathrm{X})$.
7. Let X be a continuous random variable with the probability density function

$$
f(x)=C x^{2}, \quad 3 \leq x \leq 9, \quad \text { zero otherwise. }
$$

a) Find the value of $C$ that would make $f(x)$ a valid probability density function.
b) Find the probability $\mathrm{P}(\mathrm{X}<5)$.
c) Find the probability $\mathrm{P}(\mathrm{X}>7)$.
d) Find the mean of the probability distribution of X.
e) Find the median of the probability distribution of X.
8. Suppose a random variable $X$ has the following probability density function:

$$
f(x)=\cos x, \quad 0<x<\frac{\pi}{2}, \quad \text { zero otherwise }
$$

a) Find $\mathrm{P}\left(\mathrm{X}<\frac{\pi}{4}\right)$.
b) Find $\mu=E(X)$.
c) Find the median of the probability distribution of X .
9. Let X be a continuous random variable with the probability density function

$$
f(x)=6 x(1-x), \quad 0<x<1, \quad \text { zero elsewhere. }
$$

Compute $\mathrm{P}(\mu-2 \sigma<\mathrm{X}<\mu+2 \sigma)$.
10. Suppose a random variable $X$ has the following probability density function:

$$
f(x)=x e^{x}, \quad 0<x<1, \quad \text { zero otherwise. }
$$

a) Find $\mathrm{P}\left(\mathrm{X}<\frac{1}{2}\right)$.
b) Find $\mu=E(X)$.
c) Find the moment-generating function of $\mathrm{X}, \mathrm{M}_{\mathrm{X}}(t)$.
11. Let $X$ be a continuous random variable with the probability density function

$$
f(x)=\left\{\begin{array}{cc}
c|x-3|, & 0<x<8 \\
0, & \text { otherwise }
\end{array}\right.
$$

a) Find the value of $c$ that makes $f(x)$ a valid probability density function.
b) Find the probability $\mathrm{P}(\mathrm{X}<5)$.
c) Find the median of the probability distribution of X .
d) Find the mean of the probability distribution of X .
e) Find the variance of the probability distribution of X.

