## **Discussion 05**

## 1. 2.3-11 2.5-3

If the moment-generating function of X is

$$M_{X}(t) = \frac{2}{5}e^{t} + \frac{1}{5}e^{2t} + \frac{2}{5}e^{3t},$$

Find the mean, variance, and pmf of X.

2. Suppose a discrete random variable X has the following probability distribution:

$$f(0) = \frac{7}{8},$$
  $f(k) = \frac{1}{3^k},$   $k = 2, 4, 6, 8, \dots$ 

(possible values of X are even non-negative integers: 0, 2, 4, 6, 8, ...).

Recall Week 02 Discussion Problem 1 (a): this is a valid probability distribution.

- a) Find the moment-generating function of X,  $M_X(t)$ . For which values of t does it exist?
- b) Use  $M_X(t)$  to find E(X).

**3.** Suppose a discrete random variable X has the following probability distribution:

$$f(1) = \ln 3 - 1,$$
  $f(k) = \frac{(\ln 3)^k}{k!}, \quad k = 2, 3, 4, \dots$ 

(possible values of X are positive integers: 1, 2, 3, 4, ...).

Recall Week 02 Discussion Problem 1 (b): this is a valid probability distribution.

- a) Find the moment-generating function of X,  $M_X(t)$ . For which values of t does it exist?
- b) Use  $M_X(t)$  to find E(X).

**4.** Suppose the moment-generating function of X is

$$M_{\rm X}(t) = 0.1 e^{2t} + 0.3 e^{4t} + 0.6 e^{7t}.$$

a) Find  $\mu = E(X)$ . b) Find  $\sigma = SD(X)$ .

5. Suppose a discrete random variable X has the following probability distribution:

$$f(k) = P(X = k) = a^{k}, \quad k = 2, 3, 4, 5, 6, \dots,$$
 zero otherwise.

a) Find the value of *a* that makes this is a valid probability distribution.

b) Find P(X is even).

- c) Find the moment-generating function of X,  $M_X(t)$ . For which values of t does it exist?
- d) Find E(X).

6. Let X be a continuous random variable with the probability density function

$$f(x) = \frac{C}{x^4}$$
,  $x > 5$ , zero otherwise.

- a) Find the value of C that would make f(x) a valid probability density function.
- b) Find the cumulative distribution function of X,  $F(x) = P(X \le x)$ . "Hint": Should be F(5) = 0,  $F(\infty) = 1$ .
- c) Find the probability P(6 < X < 10).
- f) Find the 80th percentile of the distribution of X,  $\pi_{0.80}$ .
- g) Find the expected value of X, E(X).
- h) Find the standard deviation of X, SD(X).

7. Let X be a continuous random variable with the probability density function

 $f(x) = Cx^2$ ,  $3 \le x \le 9$ , zero otherwise.

- a) Find the value of C that would make f(x) a valid probability density function.
- b) Find the probability P(X < 5).
- c) Find the probability P(X > 7).
- d) Find the mean of the probability distribution of X.
- e) Find the median of the probability distribution of X.

8. Suppose a random variable X has the following probability density function:

$$f(x) = \cos x, \qquad 0 < x < \frac{\pi}{2}, \qquad \text{zero otherwise.}$$
  
a) Find P(X <  $\frac{\pi}{4}$ ).  
b) Find  $\mu = E(X)$ .

c) Find the median of the probability distribution of X.

9. Let X be a continuous random variable with the probability density function

f(x) = 6 x (1-x), 0 < x < 1, zero elsewhere.

Compute P( $\mu - 2\sigma < X < \mu + 2\sigma$ ).

**10.** Suppose a random variable X has the following probability density function:

$$f(x) = x e^{x}$$
,  $0 < x < 1$ , zero otherwise.

a) Find 
$$P(X < \frac{1}{2})$$
. b) Find  $\mu = E(X)$ .

c) Find the moment-generating function of X,  $M_X(t)$ .

**11.** Let X be a continuous random variable with the probability density function

$$f(x) = \begin{cases} c | x-3 |, & 0 < x < 8, \\ 0, & \text{otherwise.} \end{cases}$$

- a) Find the value of c that makes f(x) a valid probability density function.
- b) Find the probability P(X < 5).
- c) Find the median of the probability distribution of X.
- d) Find the mean of the probability distribution of X.
- e) Find the variance of the probability distribution of X.